

The Psi function and its zeros on the complex plane

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The Psi function

I defined the **Psi function** as the sum of reciprocals of factorials to the s power:

$$\psi(s) := \sum_{n=1}^{\infty} \frac{1}{(n!)^s} \quad s \in \mathbb{C} \quad (1)$$

The zeros of the Psi function on the complex plane

I found interesting studying **the zeros of the Psi function** from a computational point of view with MATLAB/Octave and Mathematica.

I formulate two conjectures:

Conjecture 1: the non-trivial zeros of the Psi function are all in the **critical strip** $0 < \text{Re}(s) \leq 1$.

Conjecture 2: the non-trivial zeros of the Psi function are all in the strip $0 < \text{Re}(s) \leq 0.73$.

A proposal for a new mathematical constant

Below is a sequence of graphs of the levels (contour) of the function $\log_{10}(\text{abs}(\psi(s)))$ for s belonging to the complex plane.

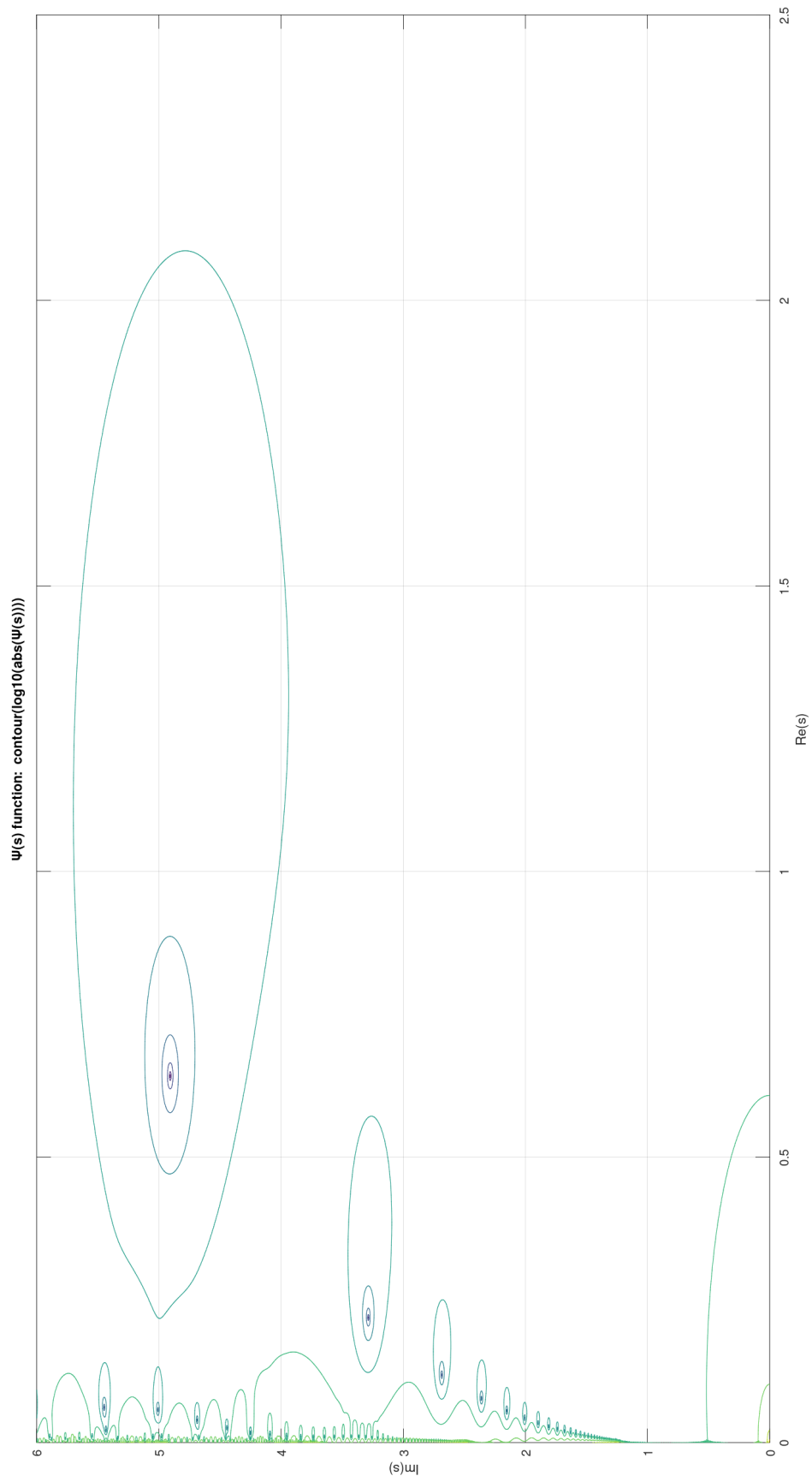
In the first graph, the **first zero in the semi-plane $\text{Re}(s) \geq \frac{1}{2}$** is in evidence.

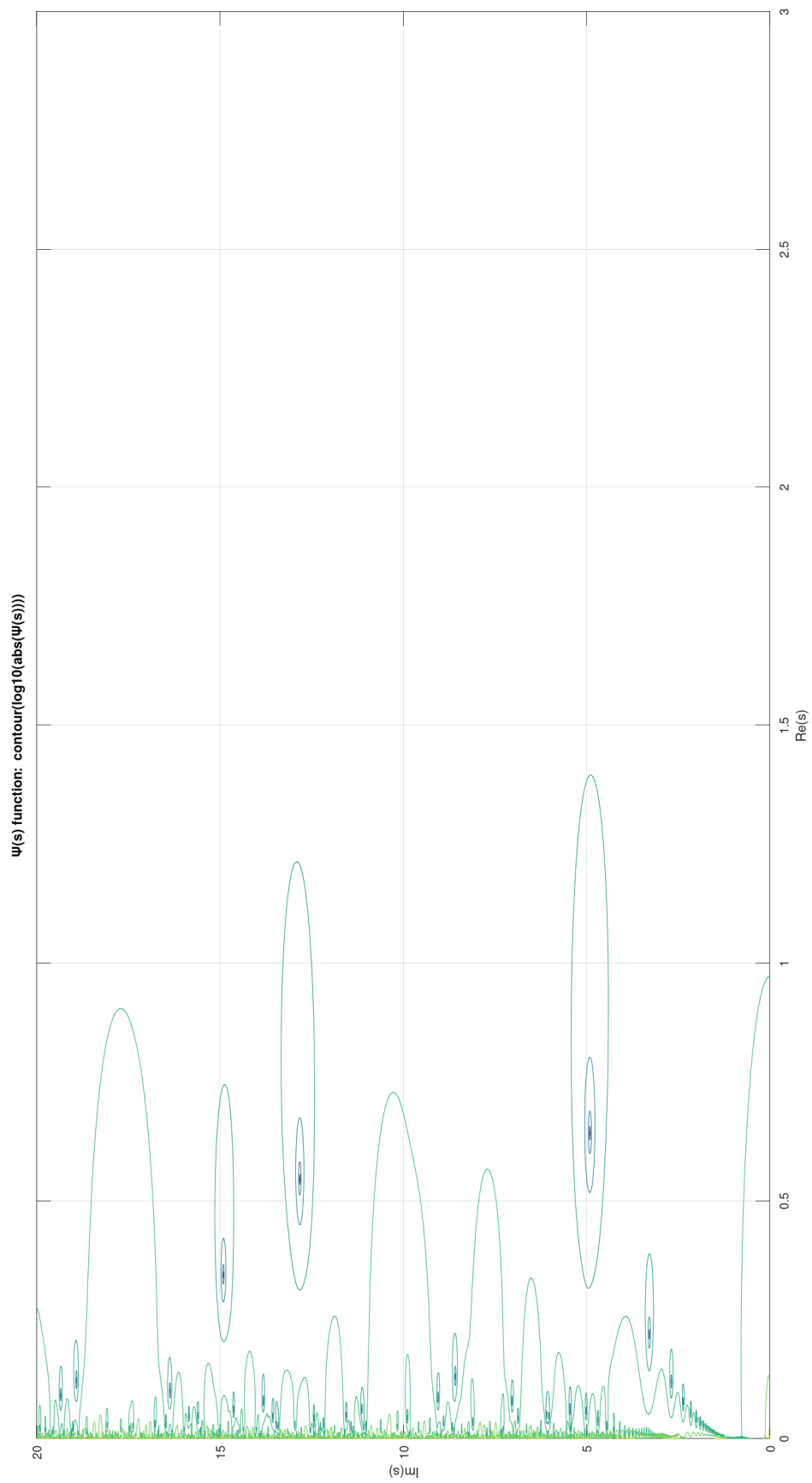
His position is approximately $s_1 \approx 0.6418158643 + 4.9068764351i$.

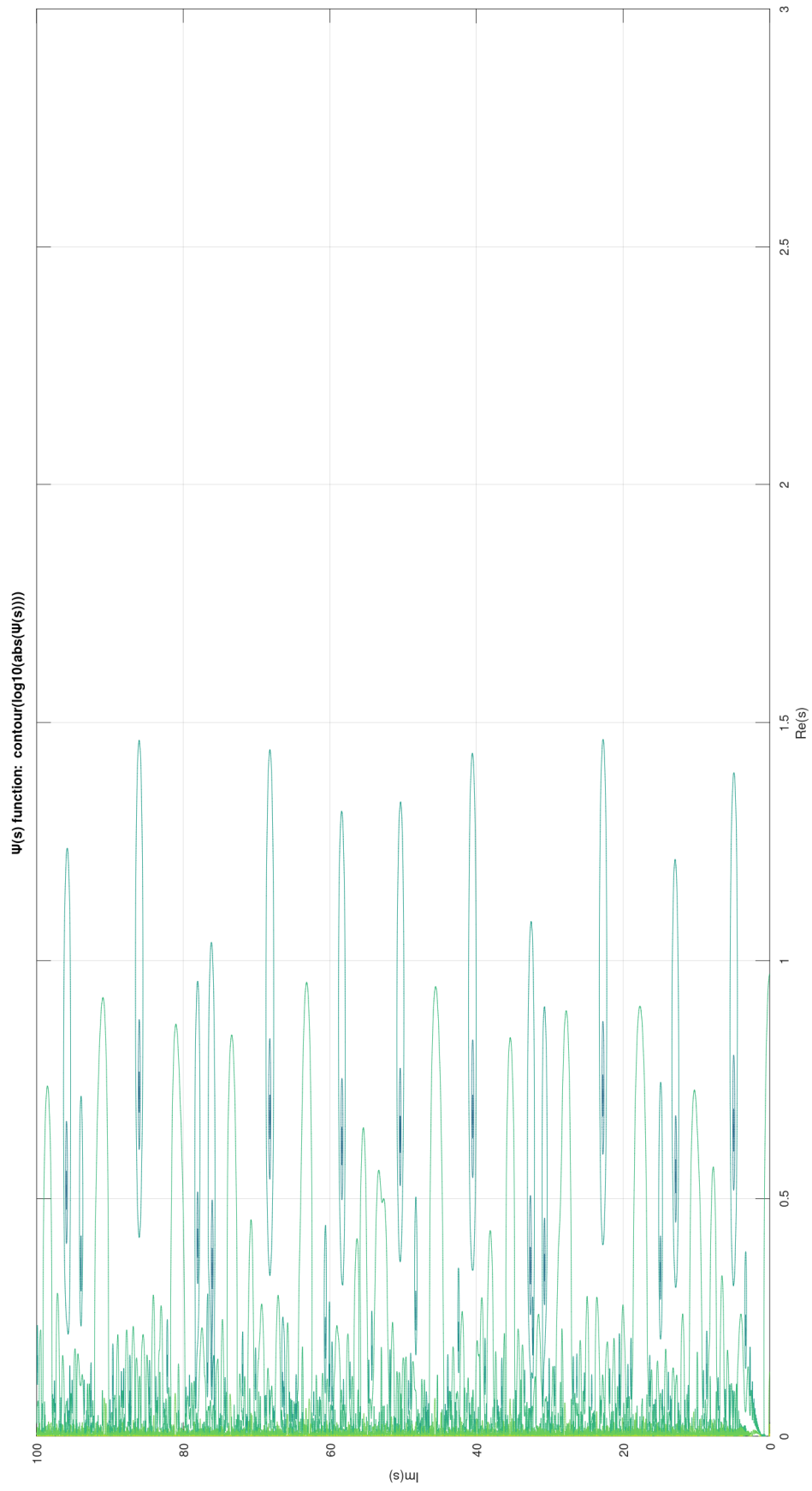
$\text{Im}(s_1) \approx 4.9068764351$ seems to be **a new mathematical constant**.

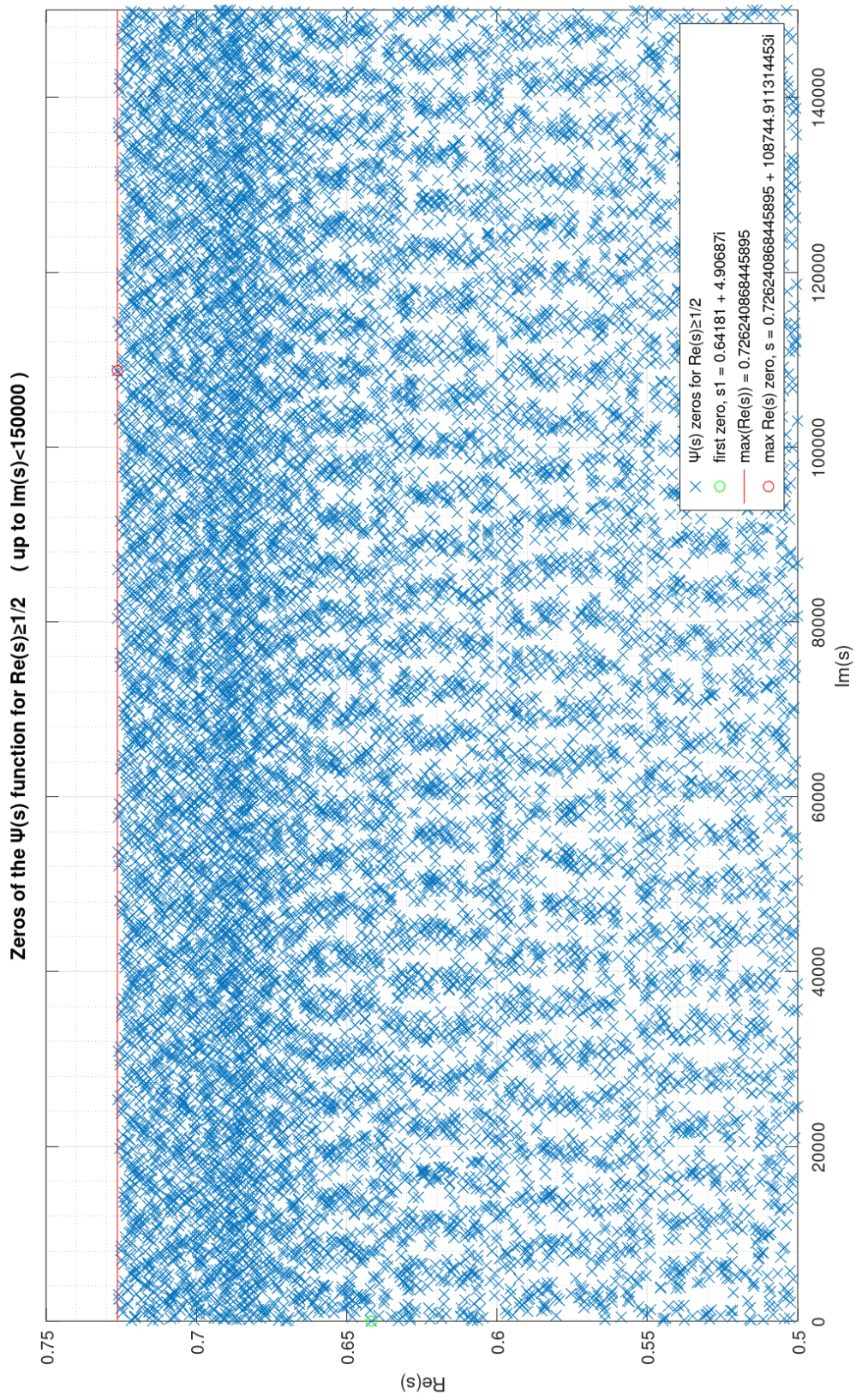
A search in the OEIS Foundation database <https://oeis.org/> is negative.

Also unsuccessful was a search for a composed constant (see at the end).





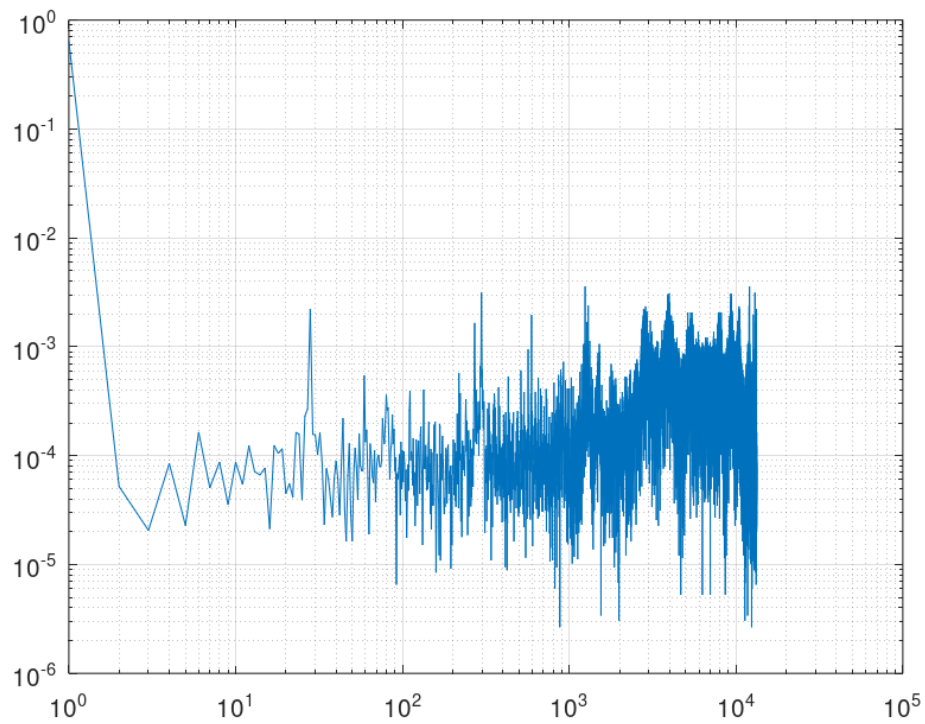




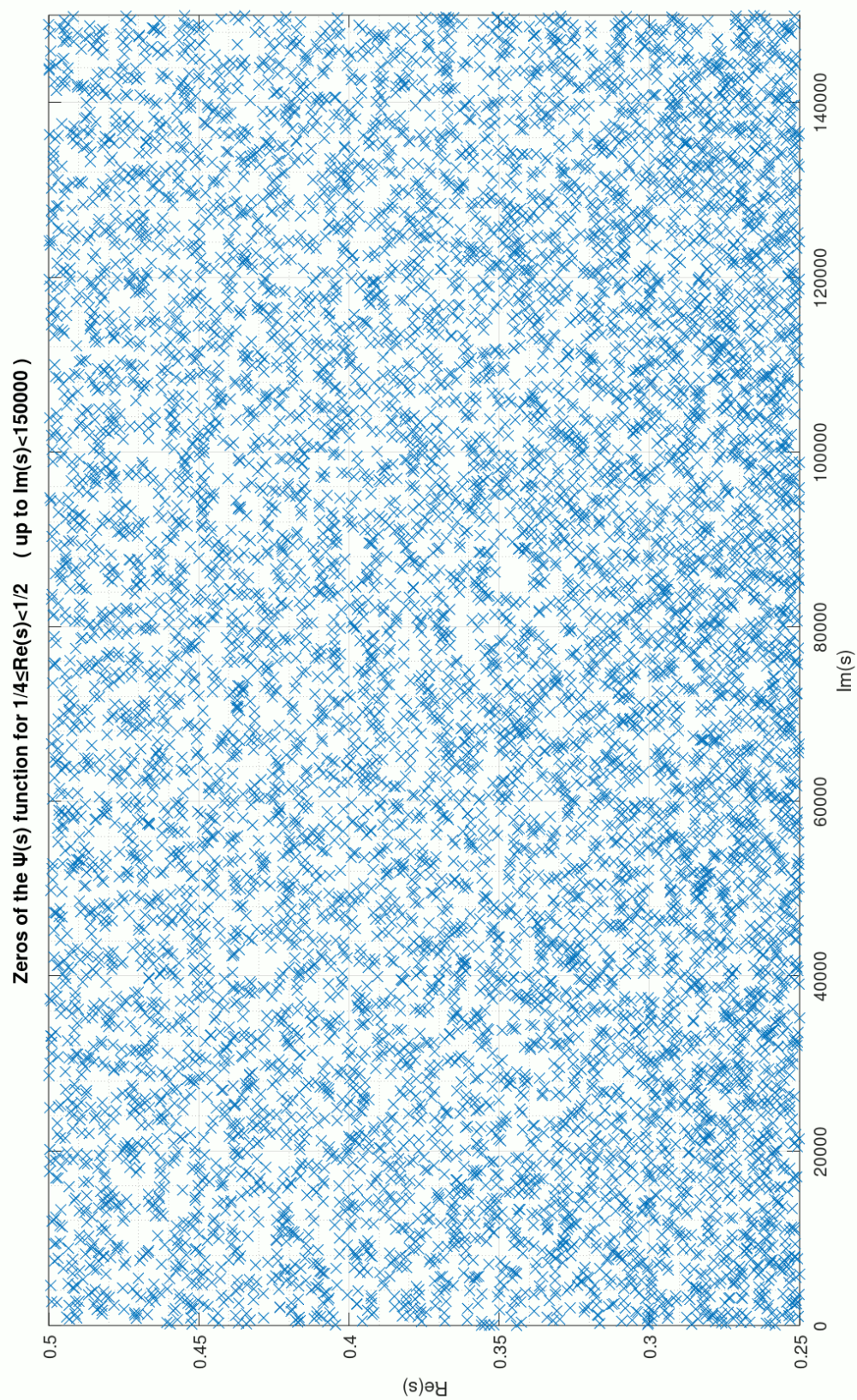
Seeing the previous graph of the first 13283 zeros ($0 \leq \text{Im}(s) \leq 150000$) for $\text{Re}(s) \geq \frac{1}{2}$, it seems that some kind of periodic patten is present in the positions of the zeros. An FFT analysis of the coordinates of the zeros reveals something similar.

Furthermore, both conjectures 1 and 2 seem true.

FFT analysis of the zeros real part (for $\text{Re}(s) \geq 1/2$)



In the successive graph there are the first 9486 zeros ($0 \leq \text{Im}(s) \leq 150000$) for the strip $\frac{1}{4} \leq \text{Re}(s) < \frac{1}{2}$.

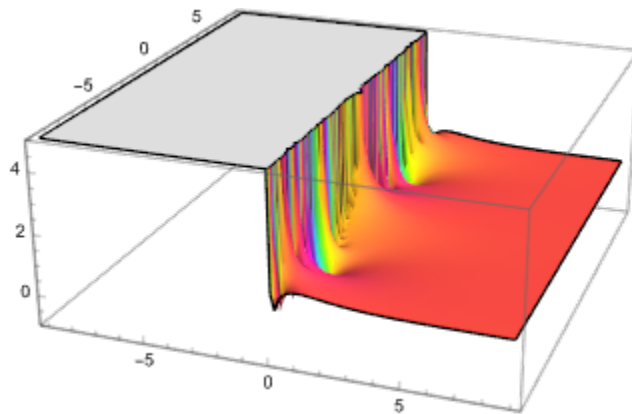


```
In[ ]:= (* Approximate Psi(s) function *)
```

```
Psi[s_, nmax_] := ParallelSum[1/n!^s, {n, 1, nmax}]
```

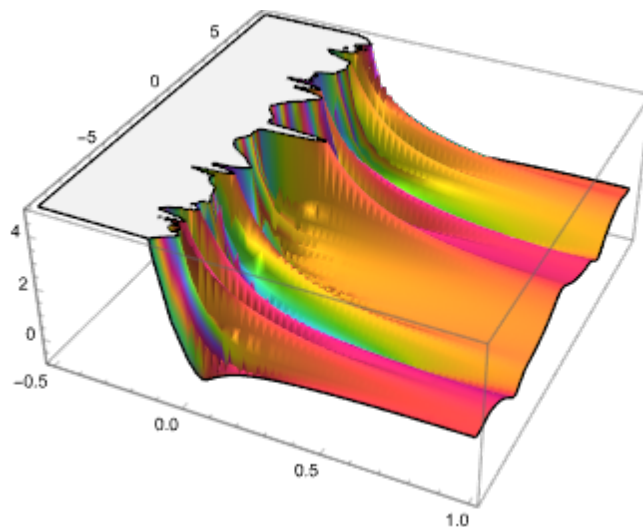
```
ComplexPlot3D[Psi[s, 10], {s, -9 - 9 I, 9 + 9 I}, PlotRange -> {-1, 5}]
```

```
Out[ ]:=
```



```
ComplexPlot3D[Psi[s, 10], {s, -1/2 - 9 I, 1 + 9 I}, PlotRange -> {-1, 5}]
```

```
Out[ ]:=
```



(* Examples of real values *)

$$\mathbf{N}[\mathbf{Psi}[1000, 200], 100] \quad (* \quad \lim_{\operatorname{Re}(s) \rightarrow \infty} \mathbf{Psi}(s) = \mathbf{1} \quad *)$$

Out[•]=

[illegible]

N[Psi[1, 200], 100] (* Psi(1) = e - 1 *)

Out[•]=

1.718281828459045235360287471352662497757247093699959574966967627724076630353547594571
382178525166427

```
In[•]:= N[Psi[1 / 2, 200], 100]
```

Out[]=

2.469506314521047562475636744660150257689756183994969199677925701971835488003408834038
662508412007135

$$In[\bullet] := N[Psi[1/4, 300], 100]$$

Out[]=

3.706322173479391073244427601647255813774406138034280338206804109345001835671964744961
441193290389664

```
In[•]:= N[Psi[5 / 1000, 20 000], 100]
```

Out[]=

58. 35677582171687522306150932691040732414573671813989447675578698932963030813328134227
727344209568700

```
In[•]:= N[Psi[1 / 1000, 20 000], 100]
```

Out[]=

212.8934344404598864421628936787745427802332320971500245174100206444701514489283865814
643740128295603

(* The s1 zero *)

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 1/2}, {y, 5}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.64181586433314113248112139394951847372318338379012,
 y -> 4.9068764351428513475351082583558535315328564648993}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 1/2}, {y, 5}}, WorkingPrecision -> 150]
```

```
Out[*]=
{x ->
  0.641815864333141132481121393949518473723183383790117174605186222762580517064969791\
  362762127145058179633534712208669467283714546798848631625316442136767, y ->
  4.906876435142851347535108258355853531532856464899337635202889524870080968491604060\
  11106519694492018106732629379978481024986065008293255799891424877982}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 1/2}, {y, 5}}, WorkingPrecision -> 200]
```

```
Out[*]=
{x ->
  0.641815864333141132481121393949518473723183383790117174605186222762580517064969791\
  3627621271450581796335347122086694672837145467988486316253164421367668138634221401\
  4473677286328546523427080926215482659, y ->
  4.906876435142851347535108258355853531532856464899337635202889524870080968491604060\
  1110651969449201810673262937997848102498606500829325579989142487798169771964786789\
  243286585094776786735969645818985011}
```

(* Other examples of zeros *)

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 2000]], Im[Psi[x + y*I, 2000]]},
  {{x, 1336 / 1000000}, {y, 1 + 4705 / 1000000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.0013361234465891396568602622971086091779190211534074,
 y -> 1.0047047214630754990772104173473400204889209870526}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 20000]], Im[Psi[x + y*I, 20000]]},
  {{x, 1336 / 1000000}, {y, 1 + 4705 / 1000000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.0013361234423282565356094057650499049578042780328027,
 y -> 1.0047047214631007857421115748599516575637423334916}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 670 / 1000}, {y, 40 + 516 / 1000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.67022547274446564754942561921853627239573382162093,
 y -> 40.516540228521088324200375633146033781651332005560}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 721 / 1000}, {y, 86 + 2 / 1000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.72159477459801208832092300307574122028151736384671,
 y -> 86.002031717653971712641931481424395869298288479074}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 200 / 1000}, {y, 512 + 787 / 1000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.19977206033939305741111784335367081276729571786498,
 y -> 512.78788254499022664698311303017535684680849482170}
```

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 611 / 1000}, {y, 1019 + 180 / 1000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.61144000424789254536789403748651182040962005969469,
 y -> 1019.1796100394148909618609953176907836874492265833}
```

0.6919053669 + 149 998.1372808481 i

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 692 / 1000}, {y, 149 998 + 137 / 1000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.69190536689323821234606604538539980100012880561469,
 y -> 149 998.13728084803537276334254075480663772664388012}
```

0.726240868445895 + 108 744.911314453 i

(* The zero with maximum Re(s) (Im(s)<150000) *)

```
In[*]:= FindRoot[{Re[Psi[x + y*I, 1000]], Im[Psi[x + y*I, 1000]]},
  {{x, 726 / 1000}, {y, 108 744 + 911 / 1000}}, WorkingPrecision -> 50]
```

```
Out[*]=
{x -> 0.72624086844586790634634422045970079610134134053575,
 y -> 108 744.91131445345894385570366150252207947684867118}
```

```

(* Factorial *)
Table[{n, n!}, {n, 1, 40}] // Column
Out[*]=
{1, 1}
{2, 2}
{3, 6}
{4, 24}
{5, 120}
{6, 720}
{7, 5040}
{8, 40320}
{9, 362880}
{10, 3628800}
{11, 39916800}
{12, 479001600}
{13, 6227020800}
{14, 87178291200}
{15, 1307674368000}
{16, 20922789888000}
{17, 355687428096000}
{18, 6402373705728000}
{19, 121645100408832000}
{20, 2432902008176640000}
{21, 51090942171709440000}
{22, 112400072777607680000}
{23, 25852016738884976640000}
{24, 620448401733239439360000}
{25, 15511210043330985984000000}
{26, 403291461126605635584000000}
{27, 10888869450418352160768000000}
{28, 304888344611713860501504000000}
{29, 8841761993739701954543616000000}
{30, 26525285981219105863630848000000}
{31, 82228386541779228172556288000000}
{32, 26313083693369353016721801216000000}
{33, 86833176188118864955181944012800000}
{34, 29523279903960414084761860964352000000}
{35, 103331479663861449296665133752320000000}
{36, 3719933267899012174679994481508352000000}
{37, 137637530912263450463159795815809024000000}
{38, 523022617466601117600072241000742912000000}
{39, 203978820811974433586402817399028973568000000}
{40, 815915283247897734345611269596115894272000000}

```

(* Some examples of contributions to the summation *)

In[]:= N[1 / (200!) ^ (1 / 2)]

Out[]:=
 3.56087×10^{-188}

In[]:= N[1 / (200!) ^ (1 / 4)]

Out[]:=
 1.88703×10^{-94}

In[]:= N[1 / (200!) ^ (1 / 1000)]

Out[]:=
0.421797

In[]:= N[1 / (2000!) ^ (1 / 1000)]

Out[]:=
 1.83857×10^{-6}

In[]:= N[1 / (20000!) ^ (1 / 1000)]

Out[]:=
 4.59981×10^{-78}

```
(* Im(s1) : Search for a composed constant *)
```

```
zero = 4.9068764351428513475351082583558535315
```

```
lista = {Pi, Pi^3, Sqrt[Pi], E, E^2, E^3, Sqrt[E], Zeta[2], Zeta[3], Zeta[4], Zeta[5],  
  1/2, 1/3, 1/4, 1/5, 1/7, 1/9, 1/11, 1/13, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13,  $\sqrt{2}$ ,  
   $\sqrt{3}$ ,  $\sqrt{5}$ ,  $\sqrt{6}$ ,  $\sqrt{7}$ ,  $\sqrt{10}$ , Erf[1], GoldenRatio, GoldenAngle, Sin[1], EulerGamma,  
  -StieltjesGamma[1], Catalan, Glaisher, Khinchin, ChampernowneNumber[10]}
```

```
lista2 = (DeleteDuplicates[Subsets[#, 6]] &)[lista]
```

```
lista3 = Exp[Total[Log[lista2], {2}]]
```

```
N[lista3, 10]
```

```
(* N[Abs[lista3-zero],6] *)
```

```
delta = lista3 - zero;
```

```
If[Min[Abs[delta]] < 0.0000001, 1000, 0]
```

```
If[Min[Abs[delta + 1]] < 0.0000001, 1000, 0]
```

```
If[Min[Abs[delta - 1]] < 0.0000001, 1000, 0]
```

```
If[Min[Abs[delta + 1/2]] < 0.0000001, 1000, 0]
```

```
If[Min[Abs[delta - 1/2]] < 0.0000001, 1000, 0]
```

Out[8]=

```
4.9068764351428513475351082583558535315
```

Out[9]=

```
{ $\pi$ ,  $\pi^3$ ,  $\sqrt{\pi}$ , e, e2, e3,  $\sqrt{e}$ ,  $\frac{\pi^2}{6}$ , Zeta[3],  $\frac{\pi^4}{90}$ , Zeta[5],  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  
 $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{7}$ ,  $\frac{1}{9}$ ,  $\frac{1}{11}$ ,  $\frac{1}{13}$ , 2, 3, 4, 5, 7, 8, 9, 10, 11, 13,  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ ,  
 $\sqrt{6}$ ,  $\sqrt{7}$ ,  $\sqrt{10}$ , Erf[1], GoldenRatio, GoldenAngle, Sin[1], EulerGamma,  
-StieltjesGamma[1], Catalan, Glaisher, Khinchin, ChampernowneNumber[10]}
```

Out[10]=

```
{ {}, { $\pi$ }, { $\pi^3$ }, { $\sqrt{\pi}$ }, {e}, ... 9531031 ...,  
{Sin[1], EulerGamma, -StieltjesGamma[1], Glaisher, Khinchin, ChampernowneNumber[10]},  
{Sin[1], EulerGamma, Catalan, Glaisher, Khinchin, ChampernowneNumber[10]},  
{Sin[1], -StieltjesGamma[1], Catalan, Glaisher, Khinchin, ChampernowneNumber[10]},  
{EulerGamma, -StieltjesGamma[1], Catalan, Glaisher, Khinchin, ChampernowneNumber[10]}}
```

Full expression not available (original memory size: 4.2 GB)



Out[11]=

```
{1,  $\pi$ ,  $\pi^3$ ,  $\sqrt{\pi}$ , e, ... 9531031 ..., -EulerGamma Glaisher Khinchin ChampernowneNumber[10] Sin[1] StieltjesGamma[1],  
Catalan EulerGamma Glaisher Khinchin ChampernowneNumber[10] Sin[1],  
-Catalan Glaisher Khinchin ChampernowneNumber[10] Sin[1] StieltjesGamma[1],  
-Catalan EulerGamma Glaisher Khinchin ChampernowneNumber[10] StieltjesGamma[1]}
```

Full expression not available (original memory size: 3 GB)



Out[12]=

```
{1.0000000000, 3.141592654, 31.00627668, 1.772453851, 2.718281828,  
... 9531031 ..., 0.01503724305, 0.1891565932, 0.02386213352, 0.01636847558}
```

Full expression not available (original memory size: 0.9 GB)



Out[13]=

```
0
```

```
Out[8]=
```

```
0
```

```
Out[9]=
```

```
0
```

```
Out[10]=
```

```
0
```

```
Out[11]=
```

```
0
```

(* Im(s1) : Continued fraction analysis *)

FindRoot[{Re[Psi[x + y*I, 2000]], Im[Psi[x + y*I, 2000]]},
{x, 642 / 1000}, {y, 4 + 907 / 1000}], WorkingPrecision -> 1300]

Out[*]=

```
{x ->
0.641815864333141132481121393949518473723183383790117174605186222762580517064969791
3627621271450581796335347122086694672837145467988486316253164421367668138634221401
4473677286328546523427080926215482659479846249916772492289981881602237283118520224
4417333055938758585385009631615835727557064789061796210218635823932399542713005035
0204285668743511452135765018402913835783627959764101832397707928329804045869673123
3952852112935835391970426094563136419697930464740539224604774622139270410310883300
7736708878687232557146057583614726268598535166340289104925111804264340286874690006
1927154737632192126718305362203835602896816405094386608490497489027429237444780496
0526525767810002287049574748726328086291789851695525285462162494026507574409428793
1970814747843750427566189358391783641205919945898239215477514139339496649281699054
6928700194707667649378066027826277900851179758388029366532542559124094344019723797
0925898648481904212149184271654322901455568241021109578630241268664621478071706202
4340142647497025049896164155059636293669416528058572846052742844744557074096343552
1771458037035516850914239273368767968885400566455305847163854703032146379788342447
7441208860865195679249633625851126577967203915388651776118255555097522165229613271
42759142316413378370835268490527767660124827408694727835504986505538354, y ->
4.906876435142851347535108258355853531532856464899337635202889524870080968491604060
1110651969449201810673262937997848102498606500829325579989142487798169771964786789
2432865850947767867359696458189850113066290730661163382959563711283636246554538625
9311006169595007178214401008312972843537525638997252224867324240802468810244364768
2269897754528785211758846762798337849792823720413343920076022017645533241309592761
8670017106975125537969090901282059969946969312936208257707365556006004179229465530
1531232642872112464958250438382077289092265651034942633975624239680074208958726094
7534108056061906906652783814127594510653026500850373634876100066307298108131651221
2681275478633034411846669135671790444853471372638060302960421000405484683877142583
6970786349664824147663633944854393850500487390514241108209352915338035876396883306
9499268132163409237863433694233007299942330799338179060919513105927809942622385429
5990204032347432446554574335708979998882199510715783327002681548879243605424013369
9741905847329101368437182707601454192408809933135631089196893186880746998141384100
7119434361379653751954961807215197669873885843532773234817736563385354967922182825
3366020757153655341617928906793558384781839689666936592212099246055045083010654624
0826746256595765893238318595226963854794396575502704864042806084344431}
```

```
In[*]:= zero = %[[2]] [[2]]
```

```
Out[*]=
```

```
4.906876435142851347535108258355853531532856464899337635202889524870080968491604060111 :
065196944920181067326293799784810249860650082932557998914248779816977196478678924328 :
658509477678673596964581898501130662907306611633829595637112836362465545386259311006 :
169595007178214401008312972843537525638997252224867324240802468810244364768226989775 :
452878521175884676279833784979282372041334392007602201764553324130959276186700171069 :
751255379690909012820599699469693129362082577073655560060041792294655301531232642872 :
112464958250438382077289092265651034942633975624239680074208958726094753410805606190 :
690665278381412759451065302650085037363487610006630729810813165122126812754786330344 :
118466691356717904448534713726380603029604210004054846838771425836970786349664824147 :
663633944854393850500487390514241108209352915338035876396883306949926813216340923786 :
343369423300729994233079933817906091951310592780994262238542959902040323474324465545 :
743357089799988821995107157833270026815488792436054240133699741905847329101368437182 :
707601454192408809933135631089196893186880746998141384100711943436137965375195496180 :
721519766987388584353277323481773656338535496792218282533660207571536553416179289067 :
935583847818396896669365922120992460550450830106546240826746256595765893238318595226 :
963854794396575502704864042806084344431
```

```
In[*]:= ContinuedFraction[zero, 1000]
```

```
Out[*]=
```

```
{4, 1, 9, 1, 2, 1, 4, 1, 1, 1, 5, 8, 28, 2, 3, 1, 1, 2, 2, 3, 3, 3, 1, 11, 3, 2, 2, 3, 1, 4, 10,
4, 3, 2, 1, 1, 1, 1, 6993, 1, 3, 1, 1, 6, 5, 3, 4, 1, 2, 5, 1, 3, 3, 4, 5, 6, 3, 15, 17,
2, 1, 130, 1, 2, 9, 1, 5, 1, 4, 8, 2, 1, 1, 4, 2, 2, 1, 7, 1, 1, 1, 1, 1, 10, 1, 4, 1, 1,
1, 6, 2, 1, 34, 1, 1, 2, 1, 1, 13, 2, 6, 7, 5, 1, 1, 1, 3, 1, 2, 9, 2, 3, 22, 1, 9, 1, 2,
1, 13, 1, 3, 2, 1, 1, 1, 1, 25, 100, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1, 2, 2, 42, 1, 10, 5, 1,
13, 2, 1, 3, 15, 1, 6, 5, 86, 4, 2, 49, 11, 1, 1, 5, 4, 2, 3, 1, 1, 4, 3, 1, 1, 4, 3, 5,
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13, 2, 1, 1, 1, 5, 6, 2, 1, 1, 9, 1, 42, 6, 14, 12, 1, 2, 2, 1, 1, 1, 4, 1, 1, 2, 5, 1, 4}
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(* Conjecture 3: irrational and transcendental number ??? *)
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Conclusions

By definition, in some mode, the **Psi function** is a “part” of the **Riemann’s Zeta function**.

The zeros of the Psi function seems to be in the **critical strip** $0 < \text{Re}(s) \leq 1$.

I proposed as **new mathematical constant** the imaginary part of the first zero of the Psi function in the semi-strip $\text{Re}(s) \geq \frac{1}{2}$: **$\text{Im}(s_1) \approx 4.9068764351$** .

I’m sorry, but I have not found reference materials really regarding a function like the Psi function and the study of its zeros.

References

- 1) <https://oeis.org/A091131>
- 2) <https://oeis.org/A058303>
- 3) <https://mathworld.wolfram.com/FactorialSums.html>
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